

# Algebraic Form of M3-Brane Action

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## Abstract

We reformulate the bosonic action of unstable M3-brane to manifest its algebraic representation. It is seen that in contrast with string and M2-brane actions that are represented only in terms of two and three dimensional Lie-algebras respectively, the algebraic form of M3-brane action is a combination of four, three and two dimensional Lie-algebras. Corresponding brackets appear as mixtures of tachyon field, space-time coordinates,  $X$ , two-form field,  $\hat{\omega}^{(2)}$ , and Born-Infeld one-form,  $\hat{b}_\mu$ .

*PACS numbers:* 11.25.Yb; 11.25.Hf

*Keywords:* M-theory; M3-brane; Lie-algebra; Nambu bracket

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## 1 Introduction

Algebraic reformulation of known actions in string theory and M-theory shows that string theory is based on conventional algebra or two dimensional Lie-algebra (known as two-algebra) but a complete description of M-theory needs an extended Lie algebra called three-algebra [1] which was mainly developed by Bagger, Lambert and Gustavsson [2–5]. Numbers two and three are associated with string theory and M-theory, respectively. Two is the string worldsheet dimension and also the codimension of D-branes in both type IIA and IIB superstring theories [6]. Three is the membrane worldvolume dimension in M-theory and the codimension of M2 and M5-branes. It means that via two-algebra interactions some Dp-branes will condense to a D(p+2)-brane [7] and through three-algebra interactions multiple M2-branes condense to a M5-brane [8–16]. These connections between two and three and respectively string theory and M-theory become obvious by rewriting Nambu-Goto actions in algebraic form.

By analogy one can expect to describe p-branes applying p+1-algebra structure [17]. These extended algebras are applied to construct worldvolume theories for multiple p-branes in terms of Nambu brackets that are classical approximations to multiple commutators of these algebras [18]. Nambu n-brackets introduce a way to understand  $n$  dimensional Lie-algebra presented by Fillipov [19]. Formulation of p-brane action in terms of p+1-algebra makes it more compact and we are left with algebraic calculations that are usually simpler to handle.

Since in string theory we are, inevitably, faced with unstable systems, study of them deepens our understanding of string theory. In bosonic string

theory the instability is always present due to tachyon presence in open string spectrum. Two examples of unstable states in superstring theories are: non-BPS branes (odd (even) dimensional branes in type IIA (IIB) theory) and brane-anti-brane pairs in both type IIA and IIB theories [20, 21]. One of the interesting facts about the dynamics of these unstable branes, generally obvious in effective action formulation, is their dimensional reduction through tachyon condensation [22–27]. During this process the negative energy density of the tachyon potential at its minimum point, cancels the tension of the D-brane (or D-branes) [28], and the final product is a closed string vacuum without a D-brane or stable lower dimensional D-branes. On the other hand stable objects in string theory can be obtained by dimensional reduction of stable branes in M-theory (M2 and M5-branes). Naturally, one can expect to have a pre-image of unstable branes in superstring theories by formulating an effective action for unstable branes in M-theory. Among different unstable systems in M-theory [29] M3-brane is noteworthy because it is directly related to M2-brane. Tachyon condensation of the M3-brane effective action results in M2-brane action and also its dimensional reduction leads to non-BPS D3-brane action in type IIA string theory [30].

Despite attempts made to formulate M3-brane action consistent with desired conditions [30] there has been no algebraic approach towards this formulation. Existence of algebraic form for the action of M2-brane, as the fundamental object of M-theory, motivated us to search for the algebraic presentation of M3-brane as the main unstable object in M-theory that its instability is due to the presence of tachyon.

What distinguishes present study from conventional algebraic formulations is instability of M3-brane. In other words, presence of tachyon and other background fields affect the resultant algebra. It is shown that pure four-algebra does not occur, as expected, and we are encountered with four, three and two-brackets that are mixtures of tachyon, spacetime coordinates and other fields.

## 2 Algebraic M3-brane action

The conventional action corresponding to a non-BPS M3-brane is a combination of DBI (Dirac-Born-Infeld) and WZ (Wess-Zumino) parts [30]

$$\begin{aligned} S &= S_{DBI} + S_{WZ}, \\ S_{DBI} &= - \int d^4 \xi V(T) |\hat{k}|^{1/2} \sqrt{-\det H_{\mu\nu}}, \\ S_{WZ} &= - \int d^4 \xi V(T) \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \partial_{\mu_1} T \hat{\kappa}_{\mu_2 \mu_3 \mu_4}, \end{aligned} \quad (2.1)$$

where  $\xi^\mu$  with  $\mu = 0, 1, 2, 3$  label worldvolume coordinates of M3-brane.  $V(T)$  is the tachyon potential which is an even function of  $T$  and is characterized as  $V(T = \pm\infty) = 0$  and  $V(T = 0) = \mathcal{T}_{M3}$  where  $\mathcal{T}_{M3}$  is M3-brane tension.  $\hat{k}^M(X)$  is the Killing vector and the Lie derivative of all target space fields vanish with respect to it [30]. Other fields in (2.1) are defined as

$$\begin{aligned} H_{\mu\nu} &= \hat{g}_{MN} \hat{D}_\mu \hat{X}^M \hat{D}_\nu \hat{X}^N + \frac{1}{|\hat{k}|} \hat{F}_{\mu\nu} + \frac{1}{|\hat{k}|} \partial_\mu T \partial_\nu T, \\ \hat{k}^2 &= \hat{k}^M \hat{k}^N \hat{g}_{MN}, \quad \hat{k}^2 = |\hat{k}|^2, \\ \hat{F}_{\mu\nu} &= \partial_\mu \hat{b}_\nu - \partial_\nu \hat{b}_\mu + \partial_\mu \hat{X}^M \partial_\nu \hat{X}^N (i_{\hat{k}} \hat{C})_{MN}, \\ \hat{D}_\mu \hat{X}^M &= \partial_\mu \hat{X}^M - \hat{A}_\mu \hat{k}^M, \quad \hat{A}_\mu = \frac{1}{|\hat{k}|^2} \partial_\mu \hat{X}^M \hat{k}_M, \\ \hat{\kappa}_{\mu_2 \mu_3 \mu_4} &= \partial_{\mu_2} \hat{\omega}_{\mu_3 \mu_4}^{(2)} - \partial_{\mu_3} \hat{\omega}_{\mu_2 \mu_4}^{(2)} + \partial_{\mu_4} \hat{\omega}_{\mu_2 \mu_3}^{(2)} \\ &\quad + \frac{1}{3!} \hat{C}_{KMN} \hat{D}_{\mu_2} \hat{X}^K \hat{D}_{\mu_3} \hat{X}^M \hat{D}_{\mu_4} \hat{X}^N + \frac{1}{2!} \hat{A}_{\mu_2} (\partial_{\mu_3} \hat{b}_{\mu_4} - \partial_{\mu_4} \hat{b}_{\mu_3}). \end{aligned} \quad (2.2)$$

The tensor  $H_{\mu\nu}$  consists of the pullback of background metric, field strength  $\hat{F}_{\mu\nu}$  of gauge field  $A_\mu$  and tachyon field,  $T$ .  $M$  and  $N$  represent spacetime indices and  $\hat{D}_\mu$  is covariant derivative. The field strength itself is expressed in terms of Born-Infeld 1-form  $\hat{b}_\mu$  and R-R sector field  $\hat{C}$ . The curvature of the 2-form  $\hat{\omega}^{(2)}$  is shown as  $\hat{\kappa}$ .

Determinant of the tensor  $H_{\mu\nu}$  in DBI action can be decomposed as

$$\sqrt{-\det H_{\mu\nu}} = \sqrt{-\det(\tilde{G}_{\mu\nu} + \tilde{F}_{\mu\nu})}, \quad (2.3)$$

where

$$\begin{aligned}\tilde{F}_{\mu\nu} &= \partial_\mu \hat{b}_\nu - \partial_\nu \hat{b}_\mu, \\ \tilde{G}_{\mu\nu} &= L_{MN} \partial_\mu X^M \partial_\nu X^N + \frac{1}{|\hat{k}|} \partial_\mu T \partial_\nu T,\end{aligned}\tag{2.4}$$

and

$$L_{MN} = g_{MN} + \frac{(i_{|\hat{k}|} \hat{C}_{MN})}{|\hat{k}|} - \frac{\hat{k}_M \hat{k}_N}{|\hat{k}|^2}.\tag{2.5}$$

Regarding (2.3), DBI action can be expanded to quadratic order [31] as

$$S_{DBI} = - \int d^4 \xi V(T) \sqrt{-\det \tilde{G}_{\mu\nu}} \left( 1 + \frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \dots \right).\tag{2.6}$$

## 2.1 DBI part of M3-brane action

To find the algebraic form of the DBI action, we start with the first term in (2.6), i.e.  $\sqrt{-\det \tilde{G}_{\mu\nu}}$ , that is determinant of a  $4 \times 4$  matrix and all its elements are sum of a tachyonic part and a space-like part ( $\partial X \partial X + \partial T \partial T$ ). This determinant is totally consisted of  $48 \times 8$  terms. These terms can be classified into sixteen  $4 \times 4$  determinants in such a way that the elements of these determinants are only  $\partial X \partial X$  or  $\partial T \partial T$  and not sum of them. So each determinant has 24 terms that summing them up leads to the same number of terms ( $16 \times 24$ ) as the initial main determinant. These 16 determinants can be categorized as: one determinant with  $\partial X \partial X$  elements (four combinations from 4 states  $\binom{4}{4} = 1$ ), one determinant with elements of  $\partial T \partial T$  ( $\binom{4}{4} = 1$ ), four determinants with three rows of  $\partial X \partial X$  elements and one row of  $\partial T \partial T$  elements ( $\binom{4}{1} = 4$ ), four determinants with three rows of  $\partial T \partial T$  elements and one row of  $\partial X \partial X$  elements ( $\binom{4}{1} = 4$ ) and finally six determinants with two rows of  $\partial T \partial T$  elements and two rows of  $\partial X \partial X$  elements ( $\binom{4}{2} = 6$ ). It is obtained that determinants with more than one row of  $\partial T \partial T$  are zero. So we are left with two kinds of determinants: a determinant consisting of only  $\partial X \partial X$  entities and those with three rows of

$\partial X \partial X$  elements and one row of  $\partial T \partial T$  entities. Since determinant does not change under exchanging of rows, by considering all possible permutations (4!) of rows for each one of the remaining determinants, the form of the four-algebra, in accordance with (A.5), emerges. At the end of the day after a tedious calculation the algebraic form of  $\sqrt{-\det \tilde{G}_{\mu\nu}}$  is obtained as

$$\begin{aligned} \sqrt{-\det \tilde{G}_{\mu\nu}} \rightarrow & \left\{ - \left( L_{MN} L_{OP} L_{QR} L_{ST} [X^M, X^O, X^Q, X^S] [X^N, X^P, X^R, X^T] \right. \right. \\ & \left. \left. + \frac{4}{|\hat{k}|} L_{MN} L_{OP} L_{QR} [T, X^M, X^O, X^Q] [T, X^N, X^P, X^R] \right) \right\}^{1/2} \quad (2.7) \end{aligned}$$

The 4-bracket of spacetime coordinates,  $X$ 's, corresponds to algebraic action derived in [1, 17] for  $p = 3$  case and with the fermionic fields turned off. The new term here is the mixed four-bracket of  $X$ 's and  $T$ .

Presenting a general algebraic form for the term  $\tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}$  in DBI action is not possible, however in some special cases it finds a simple form. For example one can consider a selfdual (anti-selfdual) field strength that corresponds to instanton. An instanton is a static (solitonic) solution to pure Yang-Mills theories [32]. They are important in both supersymmetric field theories and superstring theories mostly because of their nonperturbative effects. They also play role in M-theory for instance in applying the M2-brane actions to M5-brane [33]. The solution to field equations in Yang-Mills theory corresponding to an instanton has a selfdual (anti-selfdual) field strength [32]. Considering this property gives the following expression for  $\text{tr } \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}$  in the case of regular one-instanton solution [32]

$$\text{tr } \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} = -96 \frac{\rho^4}{((x - x_0)^2 + \rho^2)^4}, \quad (2.8)$$

where  $x_0$  and  $\rho$  are arbitrary parameters called collective coordinates.

So for the instantonic case the full algebraic form of the DBI part of the action reads as

$$\begin{aligned} S_{DBI} = & - \int d^4 \xi V(T) \left( 1 - 24 \frac{\rho^4}{((x - x_0)^2 + \rho^2)^4} \right) \\ & \times \left\{ - \left( L_{MN} L_{OP} L_{QR} L_{ST} [X^M, X^O, X^Q, X^S] [X^N, X^P, X^R, X^T] \right. \right. \\ & \left. \left. + \frac{4}{|\hat{k}|} L_{MN} L_{OP} L_{QR} [T, X^M, X^O, X^Q] [T, X^N, X^P, X^R] \right) \right\}^{1/2}. \quad (2.9) \end{aligned}$$

## 2.2 WZ part of M3-brane action

The integrand of WZ action in (2.1) can be divided into three parts by replacing  $\hat{\kappa}$  from (2.2)

$$\begin{aligned}
S_{WZ} &\rightarrow \varepsilon^{\mu_1\mu_2\mu_3\mu_4} \partial_{\mu_1} T \hat{\kappa}_{\mu_2\mu_3\mu_4} \\
&= \varepsilon^{\mu_1\mu_2\mu_3\mu_4} \partial_{\mu_1} T \left( \partial_{\mu_2} \hat{\omega}_{\mu_3\mu_4}^{(2)} - \partial_{\mu_3} \hat{\omega}_{\mu_2\mu_4}^{(2)} + \partial_{\mu_4} \hat{\omega}_{\mu_2\mu_3}^{(2)} \right. \\
&\quad + \frac{1}{3!} \hat{C}_{KMN} \hat{D}_{\mu_2} \hat{X}^K \hat{D}_{\mu_3} \hat{X}^M \hat{D}_{\mu_4} \hat{X}^N \\
&\quad \left. + \frac{1}{2!} \hat{A}_{\mu_2} (\partial_{\mu_3} \hat{b}_{\mu_4} - \partial_{\mu_4} \hat{b}_{\mu_3}) \right), \tag{2.10}
\end{aligned}$$

and each part is dealt with separately.

By expanding the first part, three terms of  $\hat{\omega}^{(2)}$  derivatives, and considering all possible permutations of four-dimensional Levi-Civita symbol,  $\varepsilon^{\mu_1\mu_2\mu_3\mu_4}$ , we come to a view of two-algebra. The reason is that according to (A.5) having two derivative factors signals a two-algebra which carries its two-dimensional Levi-Civita symbol. But since here only different permutations of  $\varepsilon^{\mu_1\mu_2\mu_3\mu_4}$  give correct signs to the terms, multiplying the resultant two-algebra by another two-dimensional Levi-Civita symbol and using the relation

$$\varepsilon^{\alpha\beta} \varepsilon_{\gamma\delta} = \delta_{\gamma}^{\alpha} \delta_{\delta}^{\beta} - \delta_{\delta}^{\alpha} \delta_{\gamma}^{\beta},$$

conduct us to the correct form. So the first part of WZ action is reformulated in terms of two-bracket as

$$\begin{aligned}
S_{WZ,1} &\rightarrow \varepsilon^{\mu_1\mu_2\mu_3\mu_4} \partial_{\mu_1} T (\partial_{\mu_2} \hat{\omega}_{\mu_3\mu_4}^{(2)} - \partial_{\mu_3} \hat{\omega}_{\mu_2\mu_4}^{(2)} + \partial_{\mu_4} \hat{\omega}_{\mu_2\mu_3}^{(2)}) \\
&= 3 \varepsilon^{\mu_1\mu_2\mu_3\mu_4} \varepsilon_{\mu_1\mu_2} [T, \omega_{\mu_3\mu_4}]. \tag{2.11}
\end{aligned}$$

In the second part of WZ action, three  $X$  derivatives,  $\partial X$ , and one tachyon derivative,  $\partial T$ , appear in a way that obviously form a four-algebra

$$\begin{aligned}
S_{WZ,2} &\rightarrow \frac{1}{3!} \hat{C}_{KMN} \varepsilon^{\mu_1\mu_2\mu_3\mu_4} \partial_{\mu_1} T \hat{D}_{\mu_2} \hat{X}^K \hat{D}_{\mu_3} \hat{X}^M \hat{D}_{\mu_4} \hat{X}^N \\
&= \frac{1}{3!} \hat{C}_{KMN} \varepsilon^{\mu_1\mu_2\mu_3\mu_4} \left( 1 - \frac{\hat{k}^P \hat{k}_P}{|\hat{k}|^2} \right)^3 \partial_{\mu_1} T \partial_{\mu_2} X^K \partial_{\mu_3} X^M \partial_{\mu_4} X^N \\
&= \frac{1}{3!} \hat{C}_{KMN} \left( 1 - \frac{\hat{k}^P \hat{k}_P}{|\hat{k}|^2} \right)^3 [T, X^K, X^M, X^N]. \tag{2.12}
\end{aligned}$$

Substituting  $A_\mu$  in the last part of WZ action we are faced with terms consisting of  $\partial X$ ,  $\partial T$  and  $\partial b$  that according to (A.5) indicate a three-algebra. Similar to the argument made for the first part of WZ action, multiplying this three-bracket by a three-dimensional Levi-Civita symbol and using the identity

$$\varepsilon^{\alpha\beta\gamma}\varepsilon_{\delta\eta\lambda} = \delta_\delta^\alpha(\delta_\eta^\beta\delta_\lambda^\gamma - \delta_\lambda^\beta\delta_\eta^\gamma) - \delta_\eta^\alpha(\delta_\delta^\beta\delta_\lambda^\gamma - \delta_\lambda^\beta\delta_\delta^\gamma) + \delta_\lambda^\alpha(\delta_\delta^\beta\delta_\eta^\gamma - \delta_\eta^\beta\delta_\delta^\gamma),$$

give the convenient three-algebra. Different permutations of four-dimensional Levi-Civita symbol are responsible for correct signs of different terms in three-algebra. So the algebraic form of this part is

$$\begin{aligned} S_{WZ,3} &\rightarrow \frac{1}{2!}\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\partial_{\mu_1}T\hat{A}_{\mu_2}(\partial_{\mu_3}\hat{b}_{\mu_4} - \partial_{\mu_4}\hat{b}_{\mu_3}) \\ &= \frac{\hat{k}_M}{2!|\hat{k}|^2}\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\varepsilon_{\mu_1\mu_2\mu_3}[T, X^M, b_{\mu_4}]. \end{aligned} \quad (2.13)$$

Therefore WZ action of M3-brane is presented in terms of two, three and four-brackets as

$$\begin{aligned} S_{WZ} &= - \int d^4\xi V(T) \left\{ 3\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\varepsilon_{\mu_1\mu_2}[T, \omega_{\mu_3\mu_4}] \right. \\ &\quad + \frac{1}{3!}C_{KMN} \left( 1 - \frac{\hat{k}^P\hat{k}_P}{|\hat{k}|^2} \right)^3 [T, X^K, X^M, X^N] \\ &\quad \left. + \frac{\hat{k}_M}{2!|\hat{k}|^2}\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\varepsilon_{\mu_1\mu_2\mu_3}[T, X^M, b_{\mu_4}] \right\}. \end{aligned} \quad (2.14)$$

It is seen that tachyon field couples with spacetime coordinates, Born-Infeld one-form  $\hat{b}_\mu$  and two-form  $\hat{\omega}^{(2)}$  through four, three and two-brackets, respectively.

### 3 Summary and conclusion

In this article we presented an algebraic form for bosonic M3-brane action by reformulating this action in terms of brackets. Since in the literature p-branes are described by p+1-algebra [17] one expects a four-algebra structure for M3-brane. But it was shown that the algebraic representation of M3-brane is a combination of four, three and two-algebras. Generally this difference stems



from the instability of the system that tachyon is responsible for. Except of a four-bracket of spacetime coordinates in DBI part, tachyon field is present in all other brackets and forms four, three and two-brackets with spacetime coordinates, two-form,  $\hat{\omega}^{(2)}$ , and Born-Infeld one-form  $\hat{b}_\mu$ , respectively. In future we try to study the dimensional reduction of this algebraic action.

## A Fillipov n-Lie algebra

Fillipov n-Lie algebra [19] as a natural generalization of a Lie algebra is defined by n-bracket satisfying the totally antisymmetric property

$$[X_1, \dots, X_i, \dots, X_j, \dots, X_n] = -[X_1, \dots, X_j, \dots, X_i, \dots, X_n], \quad (\text{A.1})$$

and the Leibniz rule

$$[X_1, \dots, X_{n-1}, [Y_1, \dots, Y_n]] = \sum_{j=1}^n [Y_1, \dots, [X_1, \dots, X_{n-1}, Y_j], \dots, Y_n]. \quad (\text{A.2})$$

n-Lie algebra is equipped with an invariant inner product

$$\langle X, Y \rangle = \langle Y, X \rangle, \quad (\text{A.3})$$

as well as the invariance under the n-bracket transformation

$$\langle [X_1, \dots, X_{n-1}, Y], Z \rangle + \langle Y, [X_1, \dots, X_{n-1}, Z] \rangle = 0. \quad (\text{A.4})$$

When  $n = 2$  the definition reduces to the usual Lie algebra and the inner product can be given by "Trace".

n-Lie algebra can be realized in terms of Nambu n-bracket defined over functional space on an n-dimensional manifold [18]

$$[X_1, X_2, \dots, X_n] \Leftrightarrow \{X_1, X_2, \dots, X_n\}_{N.B} := \frac{1}{\sqrt{\mathcal{G}}} \epsilon^{l_1 l_2 \dots l_n} \partial_{l_1} X_1 \partial_{l_2} X_2 \dots \partial_{l_n} X_n. \quad (\text{A.5})$$

$\mathcal{G}$  is determinant of the metric of the manifold and can be chosen arbitrarily since the properties (A.1)-(A.4) hold irrespective of the presence of the local factor [1].

## References

- [1] K. Lee and J. H. Park, JHEP 0904:012,2009, [arXiv:0902.2417 [hep-th]].
- [2] J. Bagger and N. Lambert, Phys. Rev. D 75 (2007) 045020 [arXiv:hep-th/0611108].
- [3] J. Bagger and N. Lambert, Phys. Rev. D 77 (2008) 065008 [arXiv:0711.0955 [hep-th]].
- [4] J. Bagger and N. Lambert, JHEP 0802 (2008) 105 [arXiv:0712.3738 [hep-th]].
- [5] A. Gustavsson, Nucl. Phys. B 811 (2009) 66 [arXiv:0709.1260 [hep-th]].
- [6] J. Polchinski, String theory. Vol. 2: Superstring theory and beyond Cambridge, UK: Univ. Pr. (1998) 531 p
- [7] R. C. Myers, JHEP 9912 (1999) 022 [arXiv:hep-th/9910053].
- [8] P. M. Ho and Y. Matsuo, JHEP 0806, 105 (2008) [arXiv:0804.3629 [hep-th]].
- [9] P. M. Ho, Y. Imamura, Y. Matsuo and S. Shiba, JHEP 0808, 014 (2008) [arXiv:0805.2898 [hep-th]].
- [10] C. Krishnan and C. Maccaferri, JHEP 0807, 005 (2008) [arXiv:0805.3125 [hep-th]].
- [11] I. Jeon, J. Kim, N. Kim, S.W. Kim and J.-H. Park, JHEP 0807 (2008) 056 [arXiv:0805.3236 [hep-th]].
- [12] J.-H. Park and C. Sochichiu, arXiv:0806.0335 [hep-th].
- [13] I. A. Bandos and P. K. Townsend, arXiv:0806.4777 [hep-th].
- [14] I. A. Bandos and P. K. Townsend, JHEP 0902 (2009) 013 [arXiv:0808.1583 [hep-th]].
- [15] I. Jeon, J. Kim, N. Kim, B. H. Lee and J.-H. Park, arXiv:0809.0856 [hep-th].

- [16] K. Lee, S. Lee and J.-H. Park, JHEP 0811 (2008) 014 [arXiv:0809.2924 [hep-th]].
- [17] D. Kamani, J. Exp. Theor. Phys. 139(2011)910, [arXiv:0904.2721v3 [hep-th]].
- [18] Y. Nambu, Phys. Rev. D 7 2405 (1973).
- [19] V. T. Filippov, Sib. Mat. Zh., 26, No 6, 126-140.
- [20] A. Sen, JHEP 08 (1998) 012 , [arXivP: hep-th/9805170].
- [21] A. Sen, NATO Science Series, 2001, Volume 564 , [arXiv:hep-th/9904207].
- [22] D. Kutasov, M. Marino, and G. Moore, JHEP 10 (2000) 045, [arXiv:hep-th/0009148]. .
- [23] T. Lee, Phys. Rev. D 64, 106004 (2001), [arXiv:hep-th/0105115].
- [24] K. Hashimoto, P. M. Ho, and J. E. Wang, Mod. Phys. Lett. A 20, 79 (2005), [arXiv:hep-th/0411012].
- [25] A. Lerda and R. Russo, Int. J. Mod. Phys. A 15, 771 (2000), [arXiv:hep-th/9905006].
- [26] Z. Rezaei and D. Kamani, J. Exp. Theor. Phys. 113:956-962, 2011, [arXiv:1106.2097 [hep-th]].
- [27] Z. Rezaei, Phys. Rev. D 85, 086011 (2012), [arXiv:1205.0120 [hep-th]]
- [28] A. Sen, Int. J. Mod. Phys. A 14, 4061 (1999), [arXiv:hep-th/9902105].
- [29] K. Intriligator, M. Kleban and J. Kumar, JHEP 02 (2001) 023, [arXiv:hep-th/0101010]. .
- [30] J. Kluson, Phys. Rev. D 79, 026001 (2009), [arXiv:0810.0585 [hep-th]].
- [31] K. Becker, M. Becker and J. Schwarz, String theory and M-theory, Cambridge University Press, 2007.
- [32] S. Vandoren and P. Van Nieuwenhuizen, arXiv:0802.1862v1.
- [33] C. S. Chu and H. Isono, Instanton String and M-Wave in Multiple M5-Branes System, [arXiv:1305.6808 [hep-th]].